A Local Search Based Approach to Solve Continuous DCOPs

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20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021)

AAMAS 2021 LONDON, UK

Introduction

Distributed Constraint Optimization Problems (DCOPs) are a powerful framework to model cooperative Multi-agent Systems. This framework has been applied to various areas of multi-agent coordination, such as:

- Distributed meeting scheduling.
- Sensor networks.
- Smart grids.

and many more.

Definition of a DCOP

A DCOP is defined as a tuple $\langle A, X, D, F, \alpha \rangle$, where,

- A $= \{a_1, a_2, ..., a_n\}$ is a finite set of agents.
- X $= \{x_1, x_2, ..., x_m\}$ is a finite set of discrete decision variables.
- D $= \{D_1, D_2, ..., D_m\}$ is a set of finite discrete domains where each D_i corresponds to the domain of variable x_i.
- F $= \{f_1, f_2, ..., f_k\}$ is a finite set of cost functions, with each $f_i : \prod_{x_j \in x^i} D_j \rightarrow R$ defined over a set of variables $x^i \subseteq X$.
- $\alpha : X \rightarrow A$ is a mapping function, which associates each variable $x_i \in X$ to an agent $a_i \in A$.

Objective of a DCOP:

$$X^* = \underset{X}{\operatorname{argmin}} \sum_{i=1}^{k} f_i(x^i) \tag{1}$$

Continuous DCOPs (C-DCOPs)

The traditional DCOP model is based on an assumption; that is, the variables are discrete decision variables. Nevertheless, a number of applications can be best modeled with continuous valued variables, such as:

- Target tracking sensor orientation.
- Cooperative air and ground surveillance.
- Network coverage using low duty-cycled sensors.

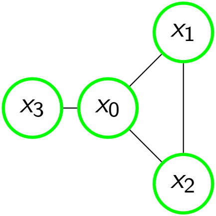
and many more.

Definition of a C-DCOP

A Continuous DCOP (C-DCOP) can also be described by a tuple $\langle A, X, D, F, \alpha \rangle$, where A, F, and α are exactly the same as those in a DCOP. X and D are defined as follows:

- X $= \{x_1, x_2, ..., x_m\}$ is a finite set of continuous decision variables.
- D $= \{D_1, D_2, ..., D_m\}$ is a set of continuous domains. Each variable x_i can choose any value from a range, $D_i = [LB_i, UB_i]$.

Example of a C-DCOP

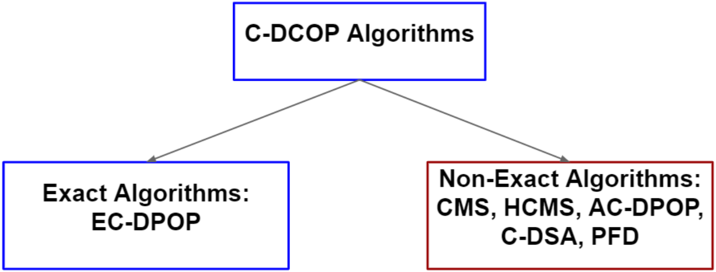


(a) Constraint Graph

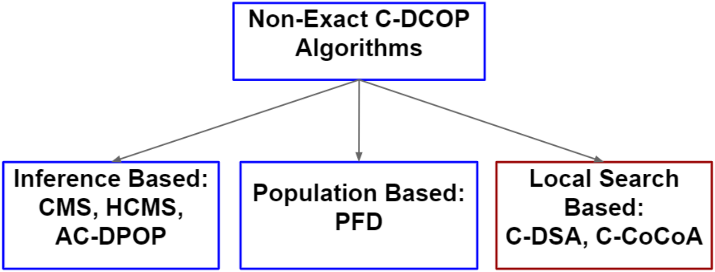
$$\forall x_i \in X : D_i = [-20, 20]$$
$$f(x_0, x_1) = x_0^2 - 2x_0x_1 + 2x_1^2$$
$$f(x_0, x_2) = x_0x_2 + 3x_2^2$$
$$f(x_0, x_3) = x_0x_3 + x_3^2$$
$$f(x_1, x_2) = x_1^2 - x_1x_2 + 2x_2^2$$

(b) Cost Functions

Exact & Non-exact C-DCOP Algorithms



Non-exact C-DCOP Algorithms



Overview

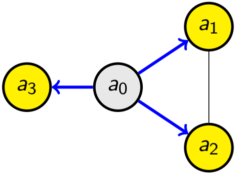
C-CoCoA is a non-iterative algorithm that is able to find high-quality solutions with a smaller communication overhead than the existing state-of-the-art C-DCOP solvers.

- Agents' states used in C-CoCoA:
  - *IDLE*: Initial state for each agent.
  - *ACTIVE*: The agent is now active and working to find an assignment for its variable.
  - *HOLD*: Variable assignment for the agent is delayed and the agent waits for more information from neighbors.
  - *DONE*: The agent has completed the assignment for its variable.

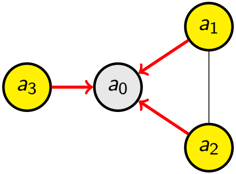
Overview

- Message types used in C-CoCoA:
    - **InquiryMessage**: Sent by an agent to its neighbors at the start of the algorithm.
    - **CostMessage**: Neighbors' reply against the **InquiryMessage**.
    - **UpdateStateMessage**: An agent updates its state.
    - **SetValueMessage**: An agent assigns a value to its variable.

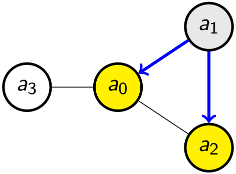
The Algorithm



(a) **InquiryMessage** from a_0



(b) **CostMessage** to a_0



(c) **InquiryMessage** from a_1

- Discretize each continuous domain into d points.
- The algorithm starts by randomly activating an agent a_i.
- a_i sends **InquiryMessage** to all the neighbors $a_j \in \mathcal{N}_i$.

The Algorithm

- Each neighbor a_j calculates the cost by using the following equation:

$$\zeta_{j,k} = \min_{x_{j,l} \in D_j} \sum_{C \in F_j} C(\widetilde{x_j} \cap x_{i,k} \cap x_{j,l}) \tag{2}$$

- Agent a_j then sends the cost map to the inquiring agent a_i.
- a_i then finds the value of its variable by using the following equation:

$$\delta = min \sum_{j=1}^{|\mathcal{N}_i|} \zeta_{j,k}; \qquad \rho = \{k : \sum_{j=1}^{|\mathcal{N}_i|} \zeta_{j,k} = \delta\} \tag{3}$$

The Algorithm

- This assignment is near-optimal within the discretized domain. In order to find the best solution within the actual continuous domain, we use a non-linear optimization technique.
- For employing the gradient-based non-linear optimization, agent a_i calculates the local objective function $F_{\mathcal{N}_i}^{a_i}$ by using the following equation:

$$F_{\mathcal{N}_i}^{a_i} = \sum_{a_j \in \mathcal{N}_i} f(a_i, a_j) \tag{4}$$

- Specifically, the agent a_i minimizes the local objective function $F_{\mathcal{N}_i}^{a_i}$ and updates the value v_x of each variable $x \in x_{\mathcal{N}_i}^{a_i}$ according to the following equation:

$$v_x(t) = v_x(t-1) - \alpha \frac{\partial F_{\mathcal{N}_i}^{a_i}}{\partial x_{\mathcal{N}_i}^{a_i}} \Bigg|_{\operatorname{argmin}_{x_i} F_{\mathcal{N}_i}^{a_i}(x_{\mathcal{N}_i}^{a_i})}^{v_x} \tag{5}$$

The Algorithm

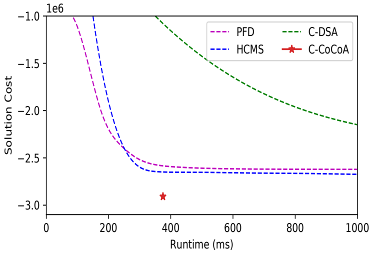
- The agent continues this update process until it converges or a maximum number of iterations is reached. After termination, the current value of v_x is actually the approximate optimal assignment for the variable x_i.
- The agent a_i then updates its state to *DONE* and communicates to its neighbors $a_j \in \mathcal{N}_i$ in a **SetValueMessage**. By receiving this message, the neighbors update their *CPA* with the value of x_i and trigger the algorithm for them.
- Note that each agent can only assign its value once and when assigned it cannot change its value. Thus C-CoCoA is a non-iterative approach.

Theoretical Analysis

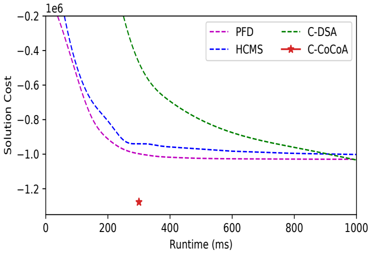
In a binary constraint graph $G = (N, E)$, in the worst case:

- The total number of messages sent or received by an agent a_i is $5|N| + d|N|$.
- The total message size for an agent a_i is $O(2|N|^2 + d|N|)$.
- The overall computational complexity is $O(|N|(d^2 + b))$.

Random C-DCOPs



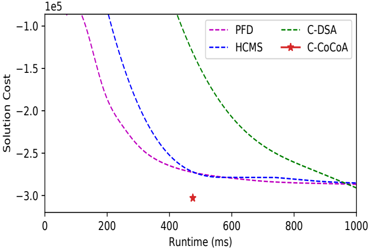
(a) Dense Graphs



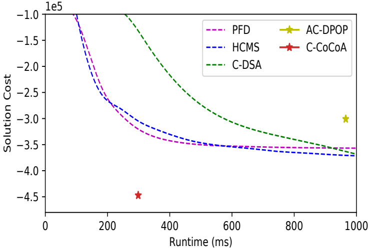
(b) Sparse Graphs

- Random C-DCOPs with 50 agents and graph density 0.2 to 0.6.
- C-CoCoA outperforms state-of-the-art non-exact algorithms by 7.97% – 24.58% on dense random C-DCOPs, 18.75% – 25.43% on sparse random C-DCOPs.

Random C-DCOPs



(a) Scale-free Graphs



(b) Random Tree

- For scale-free network, we use 100 agents.
- C-CoCoA outperforms state-of-the-art non-exact algorithms by 2.63% – 5.94% on scale-free graphs, 11.31% – 21.42% on random trees.

Random Graphs

Table: Solution quality of C-CoCoA, PFD, HCMS, and C-DSA on random graphs with varying number of agents.

		C-CoCoA	PFD	HCMS	C-DSA
$\lvert A \rvert = 30$	$p = 0.2$	-554,339	-469,093	-423,383	-493,730
	$p = 0.6$	-1,251,118	-1,042,611	-1,055,878	-1,096,852
$\lvert A \rvert = 50$	$p = 0.2$	-1,311,988	-1,030,328	-974,416	-1,118,344
	$p = 0.6$	-3,086,005	-2,623,534	-2,730,943	-2,334,385
$\lvert A \rvert = 70$	$p = 0.2$	-2,253,114	-1,858,646	-2,049,757	-1,636,508
	$p = 0.6$	-5,486,958	-4,494,102	-5,060,671	—

Conclusions & Future Work

- The classical DCOP model deals with discrete variables. But this assumption of the variables being discrete is not applicable to many real-world problems.
- In this paper, we propose a local search based algorithm, C-CoCoA, that is able to solve DCOPs with continuous variable.
- Basically, we show that a simple extension of a local search algorithm can find better solution in significantly less time than the complex inference based algorithms.
- In future, we would like to further investigate the potential of C-CoCoA on various C-DCOP applications. We would also like to explore the ways to extend C-CoCoA to solve multi-objective and asymmetric C-DCOPs.

Thank you for watching the presentation.